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**A ROBUST ESTIMATION OF INTERMITTENT DEMAND USING LOESS**

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**ABSTRACT**

Extensive research has been devoted to understanding the assumptions of Croston’s method (1972) including the impact on the demand distribution, the time between demands and the correlation between the demand size and time between demands. In practice it is not unusual with intermittent demand for the demand rate to not be constant, so being able to adapt to changing rates is necessary when forecasting this type of data. This article studies how the prevalent forecasting techniques for intermittent demand perform when a novel regression technique is used to fit the demand distribution. A modified Croston LOESS procedure is compared to Single Exponential Smoothing (SES), Croston’s method, and the Syntetos Boylan Approximation (SBA) Bias Corrected Croston Method to model the demand.

**Keywords:** forecasting, intermittent demand, Croston’s Method, LOESS

**INTRODUCTION**

Croston (1972) provided a time-tested solution to forecasting intermittent demand by creating two series for the demand size and the time between demands. The key assumptions specified by Croston for his methodology are when demand occurs the distribution of demand sizes would be normal, the distribution of time between demands would be geometric and the demand sizes and inter-arrival times are mutually independent. Shenstone and Hyndman (2005) expanded the assumptions to assume autocorrelation, non-stationary and a continuous sample space. In the Croston research attention has been paid to the interval between demands, the size of the demands and the relationship between size and interval (Willemain et al. 1994). Research has focused on providing guidance to the practitioner as to when Croston’s technique is appropriate. Demand classification methodologies discussed in the literature provide guidance on when to use a particular forecasting method, but little guidance is provided on how to adapt to changing demand patterns or outliers in the demand data.

Croston (1972) made a few basic assumptions about slow-moving data series. He assumed demand would occur as a Bernoulli process, resulting in independent and identically distributed (IID) demand resulting in a geometric distribution. Demand size was assumed to follow a normal distribution and be IID. Demand size was assumed to follow a normal distribution and be IID. The majority of the research in this area, however, does not investigate Croston’s assumption of a geometric distribution for the interval between demands. Table 1 summarizes key research that examines the performance of Croston’s method under various distributional assumptions. Following Croston’s assumptions, a geometric distribution is generally assumed for the time between demands.

**Table 1.**

**Summary of research in intermittent demand and distribution assumptions**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Demand size () | Inter-demand interval (p) | Demand Lead time |
| Croston (1972) | Normal Distribution | Geometric Distribution |  |
| Segerstedt (1994) | Normal Distribution | Geometric Distribution |  |
| Willemain et al 1994 | Normal, lognormal | Geometric Distribution |  |
| Bagchi, Havya & Ord (1983) | Normal Distribution | Geometric Distribution | Compound Distribution |
| Leven& Segerstedt (2004) | Erlang Distribution | Geometric Distribution  |  |

As shown in Table 1, the Geometric distribution is assumed to be the underlying distribution for time between positive occurrences of demand. Other articles have investigated the demand size and distributions related to the demand size and demand lead time. The authors have investigated scenarios of changing demand, that is, when demand goes from intermittent to regular demand or from regular demand to intermittent. When the demand is transitioning the assumption of a geometric distribution for the intervals between demands is questionable. Croston’s technique and SES are essentially the same when demand occurs every period, Levén and Segerstedt have suggested a variation that produces a general method for all demand levels, but potentially introduces more bias than Croston (Teunter and Sani, 2009). Levén and Segerstedt’ s (2004) methodology provides a common technique for all inventory and can be utilized for all demand types but not for an item with distinct demand rates in varying periods.

The impact that seasonality, trends, correlated demands and other issues have on forecasting items with intermittent demand has been researched (Altay, Litteral and Rudisill, 2012; Lindsey and Pavur, 2009). The authors investigated methods that adjusts to shifts in mean demand for an intermittent time series. That is, when the item would have to be categorized as slow moving and employ forecasting methodology for intermittent demand to forecast demand during the slow period and then revert to the original demand forecasting methodology when normal demand patterns return. The situation is not a seasonal adjustment since demand does not vary around seasonal averages but alternates between high or low levels.

The research presented here utilizes a novel regression technique that makes no assumptions about the parametric form of the regression surface. It is also suitable when the data set contains outliers and a robust fitting method is required. An array of simulations is conducted to determine the efficacy of a modified Croston technique that utilizes the LOESS local regression technique to model the demand function compared to traditional techniques such as SES, Croston’s method and some common modifications to Croston’s method.

**Croston’s Method**

Croston’s Method as well as the Syntetos and Boylan Approximation (SBA) are the benchmark methods for current work trying to improve forecasting for spare parts and items with intermittent demand (Singh, Simha, and Agarwal, 2024; Affonso, Conceicao and Muniz, 2024; and Turki, Jouini, Jemai, Traiy, Lazark, Valot and Heidseick, 2024). A brief description of Croston’s (1972) method will be provided in this section. The procedure is comparable to the procedure for SES. The smoothing constant  is used, and like SES it assumes a constant demand average of size of  taking place every p periods, so the mean demand is not just /p by way of Croston’s method (1972), but is instead

 y\* =  (1)

Boylan and Syntetos (2007) identify and provide a correction for a bias resulting from Croston’s method by multiplying the demand per period by 1−*α*/2. The SBA bias correction formula is (1−*α* /2) / (1−*α*/2p). Teunter and Sani (2009) point out that in cases when only a few periods have no demand, Croston’s method is better and when most periods have no demand, Syntetos and Boylan’s bias corrected or SBA technique excels. With real data, it is difficult to know if the bias corrected technique will always be beneficial. Willemain, et al (1994) methodology in implementing Croston’s method is used in this paper and is noted as follows:

 Xt = binary indicator of demand at time t

 Zt =size of demand

 Yt = XtZt = demand for an item at time t

  = mean value of demand when nonzero

 2 = variance of demand when nonzero

 *p* = average number of time periods between demands

  = smoothing parameter

 *q* = time interval since last demand

  = Croston’s estimate of mean interval between demands

  = Croston’s estimate of mean demand size

  = Croston’s estimate of mean demand per period

Additional estimates using SES are made for the average demand and the time between the demands with updates occurring if a demand occurs (Willemain et al. 1994).

If Xt = 0, Zt” = Zt-1”

 Pt” = Pt-1”

 q = q + 1 (2)

Else Xt = 1 Zt” = Zt-1” + (yt - Zt-1”)

 Pt” = Pt-1” + (q - Pt-1”)

 q = 1. (3)

The mean demand per period is then. (4)

When demand occurs, the expected value is, (5)

with the variance. (6)

Optimal results using Croston’s (1972) are achieved with alpha values between 0.1 and 0.2. Willemain, et al (1994) confirmed the guidance with respect to the alpha level. Since the data sets were designed to have no shifts in the mean and to be consistent with Croston, this work utilizes lower alpha levels. If a mean shift occurred for a data series, the forecast derived using a larger alpha would react quicker to the shift and provide a forecast with less error.

**Bias Correction**

Syntetos and Boylan determined the formulation to compute the bias correction with the true variance for the Beta Binomial distribution. The formula used in constructing the bias correction is based on a Taylor expansion of a ratio.

|  |  |
| --- | --- |
|  | (7) |

Assuming the time between demand interval series is not auto-correlated and the intervals (pt) are geometrically distributed with a mean of p and homogeneous variance of p(p-1) then:

|  |  |
| --- | --- |
| Var(x2) = Var () =  =  | (8) |

If the demand sizes are distributed with a mean “m” then equation 7 can be transformed to:

|  |  |
| --- | --- |
|  | (9) |

It follows that:

|  |  |
| --- | --- |
|  | (10) |

**LOESS Smoothing Method**

The research team of Cleveland, Develin, and Grosse (1988) developed a nonparametric method that estimates regression surfaces and makes no prior assumptions about the parametric form of the surface resulting in a very flexible procedure. When a suitable parametric form of the regression surface is not known, the LOESS procedure might be appropriate. LOESS uses local weighted regression to smooth a best fit curve in a scatter plot and identify difficult to identify trends and cycles. For any point and smoothing parameter, the algorithm finds the “k” nearest neighbors and assigns weights to perform a local weighted regression. The model is fit to local subsets to develop a function point by point. Cohen (1999) explains the LOESS method. If we assume that for i=1 to n, the ith measurement yi of the response y and the corresponding xk of the x vector of p predictors are related by

|  |  |
| --- | --- |
| Yi=g(xi) + ei | (11) |

In equation 11 “g” is the regression function and “eI"“is the random error. For the local regression near x =x0, we expect that it can be locally approximated by some function in a particular parametric class. The approximation comes from fitting the regression surface to the data in a neighborhood near x0. Weighted least squares, in the loess method, fits the linear or nonlinear functions of the predictor variable at the center of the neighborhood for a given radius. The weight decreases as they become more distant to the center of the area. This is repeated at each data point or a sample of points. By repeating the process, the model can be fitted even in the presence of outliers in the data.

The LOESS procedure is finding applications in modeling complex relationships, identifying outliers and exploratory data analysis in finance, economics, ecology and medicine. Blackledge, Webb, Nunley, Archer and Pont (2023) applied it to detect trends in COVID-19 across Texas. Krechiem and Khadir (2023) used the procedure to forecast electricity consumption in Algeria. Kumar, AnandRaj, Sreelath and Sridhar (2023) employed the methodology to study in ground water storage in India. Golubev (2024) applied the technique to studying human mortality.

**Proposed Croston and LOESS Combined Procedure**

The proposed Croston LOESS method uses SES to estimate the period between demands and the estimate of the previous demand value to estimate the next demand value. The value of Ŷt-1 is the fitted value of the LOESS method using only nonzero demand values up to the t-1 time period. The mean demand per period is computed the same as for Croston’s method. That is, the mean demand estimate at time t is the ratio of the Zt” divided by Pt”.

If Xt = 0, Zt” = Zt-1”

 Pt” = Pt-1”

 q = q + 1 (12)

Else Xt = 1 Zt” = Ŷt-1

 Pt” = Pt-1” + (q - Pt-1”)

 q = 1. (13)

**Simulation Experiment**

Simulations were conducted to assess the performance of the Croston- LOESS (CROSTL) model in addition to the traditional Croston method (CROST), the bias-corrected Croston method (SBA), and the single exponential smoothing model (SES). To judge the performance, a time series of 350 observations were generated in which the demand was considered “Fast” or “Slow” over sequential time periods. In this experiment, 50 “Fast” time periods were generated followed by 20 “Slow” time periods. Repeating the cycle five times resulted in 350 observations. The smoothing constant for the numerous models using exponential smoothing was selected to be either 0.1 or 0.2.

The first eight columns listed in Table 1 provide the conditions for each simulation. The probability of a demand was selected to be the same for the fast and the slow periods. The fast and slow designation is used mostly to distinguish between high demand periods with a larger standard deviation and low demand periods with a smaller standard deviation. The probability of a demand was assigned to be 0.4 or 0.8. The last condition in Table 2 is a benchmark condition in which the fast and slow demand periods have equal levels of mean demand with the same variation. This benchmark condition is presented in the last row and its condition is ideal for Croston’s model.

The predicted value of each model was compared to the mean for “Fast” and “Slow” periods. The root mean square (RMSE) is used to measure the accuracy of the models in estimating these means. In the last four columns, the RMSE values are provided for the accuracy of the predicted values with the actual observed values for each period. The label “ACT” is used to designate that the accuracy measure was for the prediction of the actual time period values rather than the mean values for the fast and slow periods. For each condition, 5000 simulations were generated to estimate the accuracy measures of the models. The LOESS model has a smoothing parameter that is different from that used in exponential smoothing and is set equal to 0.1 for this experiment. This value was determined by trial and error and the LOESS model generally performed more accurately for this value.

**RESULTS**

Tables 2 displays the outcomes of the four forecasting methods (SES, CROST, CROSTL, and SBA). The bolded values in the table show the minimum RMSEs for each condition. Each forecasting method performed the best in at least one condition. In the benchmark condition, which is the last condition on Table 2, as expected, the SBA procedure performed the best, followed closely by CROST. Since this last condition did not provide any changes in average demand over time, the necessary assumptions for CROST to perform optimally hold and this result is confirmed.

The proposed CROSTL method outperformed the other models in each condition in which there was a change in the “Fast” and “Slow” average demands that included a smoothing constant set to 0.1. When the smoothing constant was changed to 0.2, the SBA procedure performed the best when the probability of a demand was low, that is, the probability of demand was 0.4. Under the condition that the probability of demand was set at 0.8 and the smoothing constant was 0.2, the SES model was the best method when the mean difference in demand was equal to 400 and CROST was the best when the mean difference of across the “Fast” and “Slow” demand was equal to 200. The RMSE values generally were higher for the models when the smoothing constant was 0.2. In the literature, a smaller smoothing constant is generally recommended.

**Table 2.**

**Simulation Root Mean Square Results for Performance of Croston-LOESS, Croston, Croston Bias Corrected, and Single Exponential Smoothing models.**

|  |  |  |  |
| --- | --- | --- | --- |
|   | Mean Difference of Fast and Slow Period = 400 | Mean Difference of Fast and Slow Period = 200 | Mean Difference ... = 0 |
| LOESS Smoothing Constant | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Slow Standard Deviation | 20 | 20 | 20 | 20 | 50 | 50 | 50 | 50 | 50 |
| Fast Standard Deviation | 100 | 100 | 100 | 100 | 50 | 50 | 50 | 50 | 50 |
| Slow Mean | 100 | 100 | 100 | 100 | 300 | 300 | 300 | 300 | 300 |
| Fast Mean | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 300 |
| Alpha Smoothing | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 |
| Probability of Demand for Fast | 0.4 | 0.4 | 0.8 | 0.8 | 0.4 | 0.4 | 0.8 | 0.8 | 0.4 |
| Probability of Demand for Slow | 0.4 | 0.4 | 0.8 | 0.8 | 0.4 | 0.4 | 0.8 | 0.8 | 0.4 |
| Average RMSE SES | 83.3 | 76.8 | **110.2** | 127.1 | 76.5 | 58.5 | 76.4 | 73.4 | 34.1 |
| Average RMSE Croston | 79.8 | 80.0 | 113.7 | 133.4 | 58.4 | 49.3 | **72.1** | 74.0 | 21.8 |
| Average RMSE Croston LOESS | 80.8 | **73.3** | 113.3 | **108.0** | 58.4 | **46.4** | 72.3 | **63.2** | 26.0 |
| Average RMSE Croston SBA | **77.9** | 79.0 | 112.5 | 133.3 | **54.4** | 47.9 | 73.4 | 74.2 | **21.3** |
| Average RMSE SES Actual | 224.9 | 222.4 | **211.1** | 220.4 | 232.8 | 227.4 | 197.7 | 196.7 | 154.3 |
| Average RMSE Croston Actual | 223.4 | 223.4 | 212.9 | 224.1 | **227.3** | 225.1 | **196.2** | 196.9 | 152.0 |
| Average RMSE Croston LOESS Actual | 223.9 | **221.2** | 213.0 | **210.2** | 227.4 | **224.5** | 196.5 | **193.4** | 152.6 |
| Average RMSE Croston SBA Actual | **222.7** | 223.4 | 212.9 | 224.0 | **227.3** | 225.1 | 196.7 | 197.0 | **151.9** |

**CONCLUSIONS**

This study uses an exploratory approach to understand the potential use of LOESS models in intermittent data analysis. Many modifications of Croston’s model have been proposed, but the potential of a nonparametric technique, namely the LOESS, has not been proposed. A challenge with using the LOESS approach is that several parameters need to be assigned. The LOESS approach is a regression approach that is locally fitted. The smoothing parameter in the LOESS procedure has a different function from the smoothing parameter in exponential smoothing. The smoothing parameter in the LOESS technique indicates the fraction of data in a local neighborhood to control the smoothness of the fit. The LOESS smoothing value of 0.1 was found to be provide a better fit than higher values of the parameter for the simulated data. In exponential smoothing, as the results table indicates, a larger smoothing value tends to yield a larger RMSE value for estimating the mean demand. When comparing model performance, various smoothing values should be explored.

As seen in the results section, Croston’s method using a LOESS model for the demand performs best when the smoothing parameter is 0.1, rather than 0.2, for the exponential smoothing part of that model. The SES model in the Croston LOESS method occurs in the estimation of the average period of an occurrence. Thus, the Croston LOESS model requires two smoothing parameters to be set, one for the LOESS model and one for the exponential smoothing part to estimate the period of the occurrence of a demand.

After analyzing the benchmark Croston technique, the case for using the SBA procedure is supported but, the assumption of stationarity of a time series is a strong assumption and the assumption of strong and low levels of demand being periodic is reasonably plausible. In that case, the LOESS model can clearly be a viable alternative forecasting model in the presence of intermittent data. Further research is needed to verify its efficacy using real world data.

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