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# EFFECTS OF DELAYED DEMAND ON INTERMITTENT FORECASTING

**Matt Lindsey**
*Stephen F. Austin State University*
Nacogdoches, TX 75965
USA

**Robert Pavur**
*University of North Texas*
Denton, TX 76203-5249
USA

## ABSTRACT

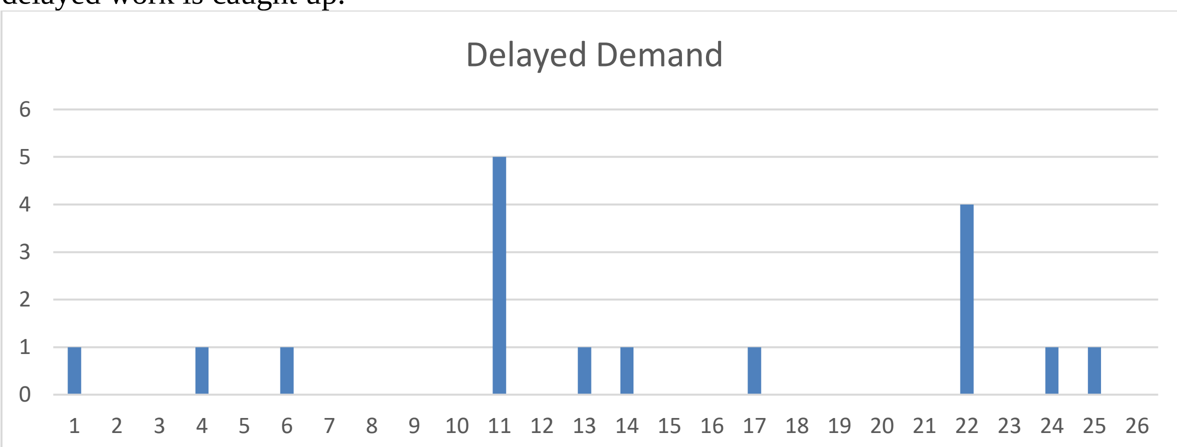
Intermittent forecasting methods, such as Croston's Method, are recommended when slow-moving demand is presence. Understanding how to classify the type of slow-moving demand may have a substantial effect on the methodology. In this study, a simulation experiment is performed to illustrate the effect of delayed demand. That is, if demand really occurs every period, but is only realized in certain periods, then there is cumulative demand. Demand is realized as the result of a sum of random variables where the number of random variables is the realization of a Poisson random variable and represents a time component. A similar approach is used in this study to generate data to gain insight into the performance of simple exponential smoothing forecast, Croston's method forecast. This study looks a type of intermittent demand that might be considered "lumpy."

**Keywords:** forecasting, intermittent demand, Croston's Method

## INTRODUCTION

Many items are classified as having intermittent demand. With positive demand on very few days or weeks or months out of the year, the robustness of most traditional forecasting procedures are challenged. In some cases, items often have steady but low demand. For some reason, when demand is not satisfied, the demand will build up over time and then be met. This pattern would be classified a "lumpy" (Syntetos, Boylan, & Croston 2005). Syntetos, Boylan and Croston (2005) identify four patterns- erratic but not very intermittent, lumpy, smooth and intermittent but not very erratic. Once a demand pattern is identified, an optimal forecasting methodology is selected. Single exponential smoothing (SES) has provided a reliable forecasting procedure for most situations when demand is stable. Forecasting with SES is also expressed as an exponentially weighted moving approach to forecasting. Croston (1972) proposed a modification to SES for cases when the demand is intermittent. The advantages of Croston's method are explained by Willemain, Smart, Shockor, and DeSautels (1994). Johnston and Boylan (1996), Syntetos and Boylan (2005) and others have demonstrated the superiority of forecasting intermittent demand with Croston's method. Seasonality, promotions, special events, trends and correlated demands can influence items with intermittent demand and research has begun to manage these effects (Altay, Litteral & Rudisill 2012; Lindsey & Pavur 2008).

A common case, often found in retail that has only been addressed in a limited manner is to adapt the forecast for periods when demand is rather constant but then is absent for a number of periods only to return at a higher level to meet pent up demand. Demand at the dollar store can diminish during the days before a pay period, only to increase on payday when delayed purchases are made. Daily maintenance is delayed for to repurpose the mechanic for another temporary job. Once the temporary job is completed, the regular maintenance schedule is resumed once the delayed work is caught up.




**Categorization**

Categorizing demand patterns to offer direction in forecasting and stock control is supported in the literature. (Syntetos, Boylan, & Croston 2005; Boylan, et al 2006). By appropriately categorizing demand, the practitioner can select the best forecasting method (Syntetos, et al 2005). The literature suggests a proper course of action exists for a given demand level, but not what the measure should be or the cutoff point within a measure.

**Aggregation Approaches**

Another approach to forecasting slow demand is aggregation. An aggregate-disaggregate intermittent demand approach (ADIDA) is proposed by Nikolopoulos, Syntetos, Boylan, Petropolulos and Assimakopoulos (2011). It can eliminate intermittent daily data by condensing it into weekly data that is forecasted and then distributed into a daily series. While the original ADIDA methodology had specific weaknesses (Petropoulos & Kourentzes 2015; Rostami-Tabar, et al 2013; Spithourakis, et al 2012 ) several variations have been suggested for specific demand patterns and address the problems with ADIDA (Petropoulos, Kourentzes, & Nikolopoulos 2016; Kourentzes, et al 2017; Athanasopoulos, et al 2017; and Li & Lim 2018.) ADIDA is not be appropriate here, since forecasting is straightforward when demand is constant. It is the few periods of zero demand that causes the difficulty in forecasting demand.

**Use of Delayed Demand in Forecasting and Stock Control**

Time can be considered a discrete variable, which allows demand to be generated using a Bernoulli process with a geometric distribution for time between demand. When realized demand follows a Poisson, demand can be modeled using a cumulative delayed demand pattern (Syntetos, Keyes & Babai 2008). Many early inventory control policies were based on delayed demand. Archibald and Silver (1978) used recursive formulations for optimal order policies. Ward (1978) used regression models based on a Poisson distribution for reorder point policy determination. Babai, Jemai and Dallery (2011) model demand as a process in which demand is carried forward and realized at some point and study an approach to compute an optimal order level for backorders. If a researcher understands the underlying demand pattern, then it may be easier to identify the most appropriate forecasting and inventory control methods. The coefficient of variance is often recommended as a means of categorizing demand patterns. Clearly, when demand follows erratic patterns, identification of an appropriate forecasting model will be challenging.

**Intermittent Demand Forecasting Method**

Croston's (1972) method is based on SES. Willemain, et al (1994) provides a procedure for applying Croston's method and should be consulted for a full explanation of the procedure. SES estimates are made for the demand mean and the time between the demands for Croston's method with updates occurring if a demand occurs. In the following equations, $X_t$ represents a positive demand with a value of one and no explicit demand with a value of zero. The $Z_t$ represents the observed demand. The symbols with the quote symbol represented smoothed values. The symbol q represents a period between actual realized demands. The $P_{t'}$ represents the smoothed value of q. The p and μ symbols represents the true mean period between demands and the true mean of the demand when it occurs. The SES method will represent simple exponential smoothing of the demand, both positive and zero. Thus, SES has variation that is more inherent in the smoothing methods. The $y_t$ estimate, illustrated below, is the estimate of the true mean demand per period.

If $X_t = 0$, $\quad Z_t'' = Z_{t-1}''$
$P_t'' = P_{t-1}''$
$q = q + 1$ (1)

Else $X_t = 1$ $\quad Z_t'' = Z_{t-1}'' + \alpha(y_t - Z_{t-1}'')$
$P_t'' = P_{t-1}'' + \alpha(q - P_{t-1}'')$
$q = 1.$ (2)

Yielding a mean demand per period of $y_t'' = \dfrac{z_t''}{p_t''}$. (3)

The expected value, when demand occurs is $E\{y_t''\} = \dfrac{\mu}{p}$, (4)

with the variance $V(y_t'') = \left[\dfrac{\alpha}{2-\alpha}\right]\left[\dfrac{(p-1)^2\mu^2}{p^4} + \dfrac{\sigma^2}{p^2}\right]$. (5)

**Simulation Experiment to Assess Benefit of Croston's Approach**

To assess the performance of the forecasting methods, both SES and Croston, demands are generated from a normally distributed population with a mean of 200 and a standard deviation of ten for 600 periods. Values for a binary variable $X_t$ are generated using a Bernoulli distribution with a certain probability of demand. The probability of an occurrence will be either 0.25, 0.50 or 0.75. These values represent data that may be considered intermittent. If the values for the demand are taken to be positive when $X_t = 1$ and are zeroed out for $X_t = 0$, then we will label this approach as the Geometric Distribution approach. Croston method has been illustrated to outperform SES in this scenario and the corrected Croston methods should provide an incremental improvement.

The second approach will be labelled as the "Delayed Demand" approach. In this approach, the demand never really goes away. It is simply accumulated until the next period in which $X_t = 1$. All of the periods in which $X_t = 0$ show zero demands. However, the first period immediately following these zero demand periods will have a demand equal to the sum of the generated demands for each period. Thus, if there were no zero demand periods, then each period would have the generated value from a normal distribution with mean of 200 and standard deviation of 10. Demand is realized as a sum of random variables with the total number of variables typically being equal to a value of a Poisson random variable. The Root Mean Squared Error (RMSE) is used to measure the accuracy of each forecasting method using data from a simulation using the geometric distribution approach and then from data generated using the a delayed demand approach. The length of each generated time series is 600 periods with the first 20 periods being ignored since they are used to start the estimation process. The RMSE is computed from 500 replications of these time series.

## RESULTS

Tables 1 and 2 provide the results of the performance of the four forecasting methods using two different smoothing constants. The geometric approach in Table 1 does not reveal new information. However, these results are included to illustrate the magnitude and pattern of the performance of Croston's procedure and its modifications. Perhaps the most interesting aspect of the Geometric approach is that the reduction in the RMSE by Croston and its modifications increase substantially as the probability of demand per period decreases.

When viewing the experimental results with delayed demand, the reduction in the RMSE by Croston's method and its modifications is substantial. As the probability of demand per period increases, the reduction decreases. Another insight is that when the smoothing constant is increased, the reduction in the RMSE does not improve by much when the Geometric approach is used. However, the reduction in the RMSE does indeed improve for the Croston method and its modifications when the smoothing constant is increased to 0.3.

## CONCLUSIONS

This study uses a scenario in which demand never really disappears or is zero but is pent-up or delayed. If the standard assumptions for demand do not hold, it is not obvious which forecasting method will perform better. Understanding when this type of data pattern occurs is a future challenge for practitioners wishing to use the appropriate intermittent forecasting procedure. Researchers has illustrated using both simulated and real-world data that forecasting approaches, such as the traditional exponentially smoothing techniques are not optimal for intermittent data. However, a theoretical question is how much one can expect Croston's method to improve the modeling of the data. The results of this study add to the extensive literature on forecasting in illustrating the degree to which Croston's method is beneficial with a delayed demand pattern.

**Table 1:**

**Accuracy of forecasting methods with smoothing constant of 0.1. RMSE for the four forecasting methods.**

| Smoothing Constant | Probability of Demand per period | Distribution | Exponential Smoothing | Croston Method | Reduction by Croston Method |
|---|---|---|---|---|---|
| 0.1 | 0.25 | Geometric | 19.82 | 10.21 | 48.50% |
| 0.1 | 0.50 | Geometric | 22.83 | 16.19 | 29.10% |
| 0.1 | 0.75 | Geometric | 19.84 | 16.68 | 15.93% |
| | | | | | |
| 0.1 | 0.25 | Delayed Demand | 62.16 | 22.59 | 63.65% |
| 0.1 | 0.50 | Delayed Demand | 27.51 | 8.21 | 70.17% |
| 0.1 | 0.75 | Delayed Demand | 13.62 | 3.18 | 76.69% |

**Table 2:**
**Accuracy of forecasting methods with smoothing constant of 0.3. RMSE for the four forecasting methods.**

| Smoothing Constant | Probability of Demand per period | Distribution | Exponential Smoothing | Croston Method | Reduction by Croston Method |
|---|---|---|---|---|---|
| 0.3 | 0.25 | Geometric | 36.37 | 20.91 | 42.51% |
| 0.3 | 0.50 | Geometric | 42.04 | 30.24 | 28.06% |
| 0.3 | 0.75 | Geometric | 36.46 | 28.77 | 21.10% |
| | | | | | |
| 0.3 | 0.25 | Delayed Demand | 163.18 | 8.63 | 94.71% |
| 0.3 | 0.50 | Delayed Demand | 80.32 | 3.44 | 95.72% |
| 0.3 | 0.75 | Delayed Demand | 41.51 | 3.66 | 91.18% |

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## ABOUT THE AUTHORS

**Matthew Lindsey** is an Associate Professor of Management and Chair of the Management and Marketing Department at Stephen F. Austin State University. He holds a PhD from the University of North Texas. His primary research area is inventory forecasting models specializing in models for items with intermittent demand, and he is active with the Decision Sciences Institute. Dr. Lindsey's primary teaching interest includes operations management and management science. Before coming to SFA he taught at the University of Texas at Tyler and the University of North Texas. In 2020 he will serve as the President of Southwest Decision Sciences Institute. He worked over 12 years in the foundry industry in various areas of production including time as tooling manager, shift superintendent and quality manager.

**Robert J. Pavur** is a Professor in the Information Technology & Decision Sciences Department at the University of North Texas. He holds a masters and doctorate degree in Statistics from Texas Tech University. He has published in such journals as the Annals of Operations Research, IEEE Transactions on Reliability, European Journal of Operational Research, Journal of the Operational Research Society, and International Journal of Operations and Quantitative Management. He is a co-author of the textbook Introduction to Business Statistics: A Computer Integrated, Data Analysis Approach published by Cengage. He is a long-time member of the Decision Sciences Institute and Southwest Decision Sciences and has held several officer positions.